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SOLIDIFICATION OF DROPLETS ON A COLD SURFACE

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Abstract—Title problem has been investigated theoretically and experimentally. In the theory a simple model of two-dimensional radial flow has been used. The degree of flattening ξ_m of a droplet depends upon the Weber, Reynolds and Péclet numbers, and upon the freezing constant U, taken from the solution of a Stefan problem. The agreement of the theory with experiments is not bad if the constant U is taken for the Stefan problem with the isothermal cooling surface.

NOMENCLATURE

A, surface covered by a droplet;
a, thermal diffusivity;
b, thickness of the liquid layer;
C, constant;
c, specific heat;
D, droplet diameter;
E_k, E_p, kinetic and potential energies;
Δh, latent heat of fusion;
k, parameter;
L_f, work of friction;
Pe, $= wD/a$, Péclet number;
Re, $= \rho' wD/\mu$, Reynolds number;
R, radius of the disk;
r, co-ordinate;
T, dimensionless temperature;
T', temperature;
t, dimensionless time;
t', time;
U, freezing constant;
V_s, volume of the solidified layer;
We, $= \rho' w^2 D/\sigma$, Weber number;
w, droplet velocity;
w_x, w_r, velocity components;
x, co-ordinate;
y, thickness of the solidified layer.

Greek symbols

α, angle;
ε, constant;
λ, heat conductivity;
μ, viscosity;
ρ, density;
σ, surface tension;
τ, dimensionless variable, or shear stress;
ξ, dimensionless radius of the disk.

1. INTRODUCTION

THE QUALITY of layers obtained by means of plasma spraying depends on the mechanism of the deformation, solidification, and adhesion of the droplets striking on the surface to be covered. The purpose of the present study is to investigate the process in order to determine the degree of flattening of the solidifying droplets, or the area A covered by a droplet of diameter D. The principal difficulty of the problem consists in the interaction of the two simultaneous processes, namely that of the motion of the liquid, and that of its solidification. Therefore in the construction of the model several simplifications must be done. Figure 1 shows a photograph of alumina (Al_2O_3) disks or "cakes" obtained in spraying of droplets of diameter $D = 2\cdot10^{-5}$–$4\cdot10^{-5}$ m impinging onto the surface with a velocity w of about $200\,\mathrm{m\,s^{-1}}$. The area covered by a droplet could be estimated, and the ratio

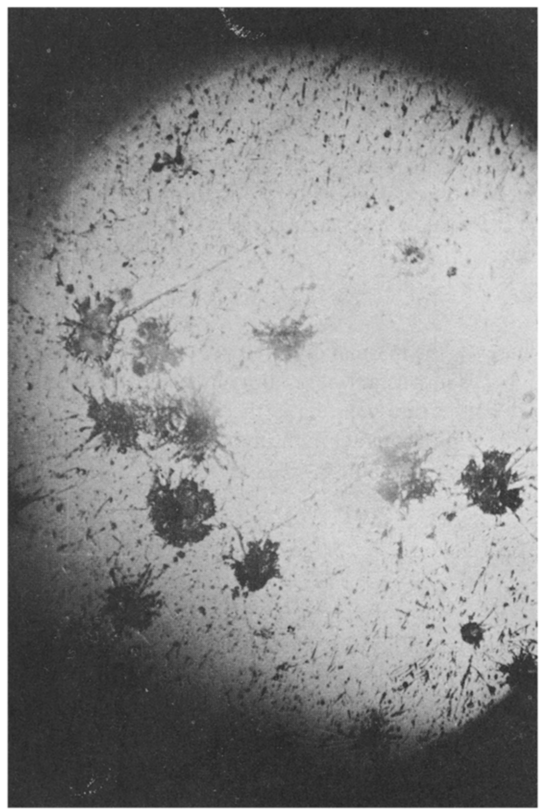

FIG. 1.

$$\xi_m = \frac{2}{D}\sqrt{\frac{A}{\pi}} \tag{1}$$

was found to be 5–6. A similar form of the disk of lead (droplet diameter $D = 3.259$ mm, $A = 174\,\mathrm{mm}^2$, $\xi_m =$

4.567, $w = 3.98\ \mathrm{m\,s^{-1}}$) is shown in Fig. 5 (specimen C10). In both cases the circumference of the disks is jagged due to droplet vibrations, but as a first attempt the form of a regular circle may be used as a model.

2. THEORY

A droplet of diameter D impinging perpendicularly on the cold surface flattens to form a cylinder of radius $R(t')$ growing with time t' (Fig. 2). Because of the

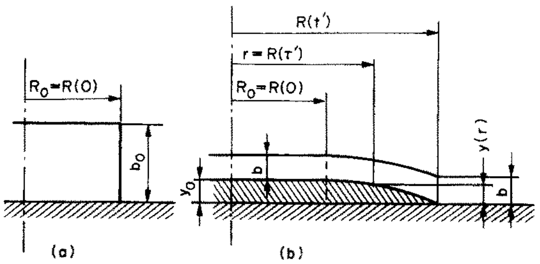

FIG. 2.

growth of the radius the thickness of the solidified layer y is greater in the centre than on the periphery. In the analogous Stefan problems this thickness is equal to

$$y = U\sqrt{(at')}, \tag{2}$$

where a is the thermal diffusivity of the solidified layer, and U is a constant depending upon the Jakob number. In the case considered there is $y = y(r)$, because the freezing begins at r in the moment τ' when $r = R(\tau')$. Therefore we assume

$$y(r) = U\sqrt{[a(t'-\tau')]} \tag{3}$$

for $r > R_0$. For $r < R_0 = R(0)$ there is

$$y_0 = U\sqrt{(at')} \tag{4}$$

as shown in Fig. 2. The volume of the solid layer is thus given by

$$V_s = \pi R_0^2 y_0 + \int_{\tau'=0}^{\tau'=t'} 2\pi R(\tau')\cdot y \cdot \mathrm{d}R(\tau') \tag{5}$$

or

$$V_s = \pi R_0^2 U\sqrt{(at')} + \int_0^{t'} 2\pi R(\tau')\frac{\mathrm{d}R(\tau')}{\mathrm{d}\tau'} U\sqrt{[a(t'-\tau')]}\,\mathrm{d}\tau'. \tag{6}$$

The next simplification consists in assumption of the liquid layer thickness b depending only upon time t', as shown in Fig. 2. Therefore the mass balance of the disk yields

$$m = \frac{\pi}{6}D^3\rho' = \rho V_s + \pi R^2 b\rho', \tag{7}$$

where m is the mass of the droplet, and ρ, ρ' are the densities of the solid and the liquid, respectively. Hence

$$b(t') = \frac{\frac{\pi}{6}D^3\rho' - \rho V_s}{\pi R^2\rho'}. \tag{8}$$

The motion or flattening of the disk (droplet) will be described by means of the energy equation

$$\frac{\mathrm{d}}{\mathrm{d}t'}(E_k + E_p + L_f) = 0, \tag{9}$$

where E_k is the kinetic energy, E_p the potential one, and L_f the work of friction forces.

To determine the kinetic energy and the work of friction the information about the velocity field is necessary. The simplest velocity distribution

$$w_x = -Cx^2, \quad w_r = Cxr, \tag{10}$$

fulfilling only the continuity equation

$$\frac{\partial}{\partial r}(rw_r) + \frac{\partial}{\partial x}(rw_x) = 0, \tag{11}$$

will be assumed. The constant C appearing above will be expressed in terms of the disk expansion $\mathrm{d}R/\mathrm{d}t'$; the latter will be assumed as the average velocity on the periphery, namely

$$\mathrm{d}R/\mathrm{d}t' = \frac{1}{b}\int_0^b w_r\,\mathrm{d}x = CbR/2, \tag{12}$$

whence

$$C = \frac{2}{Rb}\mathrm{d}R/\mathrm{d}t'. \tag{13}$$

The kinetic energy can be now calculated in this way

$$E_k = 2\pi\int_0^R r\,\mathrm{d}r\int_0^b \mathrm{d}x\cdot\tfrac{1}{2}\rho'(w_x^2 + w_r^2) \tag{14}$$

or, with use of (10) and (13)

$$E_k = \frac{\pi}{3}\rho'\left(\frac{\mathrm{d}R}{\mathrm{d}t'}\right)^2\cdot(bR^2 + \tfrac{6}{5}\cdot b^3) = E_k(t'). \tag{15}$$

The initial value is that of the falling droplet, namely

$$E_k(0) = \frac{\pi}{6}D^3\rho'\frac{w^2}{2}. \tag{16}$$

Assuming the initial radius as a portion of droplet diameter

$$R(0) = R_0 = \varepsilon D \tag{17}$$

we can calculate the initial radial expansion

$$\frac{\mathrm{d}R(0)}{\mathrm{d}t'} = w\sqrt{\left(\frac{3/2}{1+\frac{1}{30\varepsilon^6}}\right)}. \tag{18}$$

The friction power may be assumed in this form

$$\frac{\mathrm{d}L_f}{\mathrm{d}t'} = \int_0^R 2\pi r\,\mathrm{d}r\tau\overline{w_r}, \tag{19}$$

where

$$\tau = \mu\frac{\partial w_r}{\partial x} = \mu Cr = \frac{2\mu r}{Rb}\cdot\frac{\mathrm{d}R}{\mathrm{d}t'} \tag{20}$$

is the shear stress, and μ the dynamic viscosity of the liquid; also

$$\overline{w_r} = \frac{1}{b}\int_0^b w_r\,\mathrm{d}x = \frac{r}{R}\frac{\mathrm{d}R}{\mathrm{d}t'} \tag{21}$$

is the average radial velocity. Substitution of (20) and (21) into (19) yields

$$\frac{\mathrm{d}L_f}{\mathrm{d}t'} = \frac{\pi R^2 \mu}{b} \cdot \left(\frac{\mathrm{d}R}{\mathrm{d}t'}\right)^2. \tag{22}$$

The potential energy is due to the surface tension σ acting on the free surface of the disk, thus

$$E_p = \sigma(\pi R^2 + 2\pi R b). \tag{23}$$

In the initial moment $t' = 0$ this energy should be equal to the potential energy of the droplet

$$E_{pg} = \pi \sigma D^2. \tag{24}$$

However, on substituting (17), one obtains

$$E_p(0) = \pi \sigma D^2 \left(\varepsilon^2 + \frac{1}{3\varepsilon}\right). \tag{25}$$

The deficiency of the model consists in that it is not possible to assume a value of ε fulfilling the condition $E_{pg} = E_p(0)$. A discussion of this problem is given in [1]. For the further analysis the value $\varepsilon = 0.5$ is assumed, and this results in an error of 8% in the initial value of the potential energy.

Substituting now (15), (22) and (23) in (9) and introducing the dimensionless variables

$$\xi = R/R_0, \quad \varphi = b/R_0, \quad t = wt'/R_0 \tag{26}$$

as well as the Weber, Reynolds, and Péclet numbers

$$We = \rho' D w^2/\sigma, \quad Re = \rho' w D/\mu, \quad Pe = wD/a \tag{27}$$

and the parameter

$$k = 6\varepsilon^2 U \frac{\rho}{\rho'} \sqrt{\left(\frac{\varepsilon}{Pe}\right)}, \tag{28}$$

we arrive at the equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\varepsilon}{3}\dot{\xi}^2 \varphi(\xi^2 + \tfrac{6}{5}\varphi^2) + \frac{1}{We}\xi(\xi + 2\varphi)\right] + \frac{\xi^2 \dot{\xi}^2}{\varphi Re} = 0, \tag{29}$$

where

$$\varphi = \frac{1}{6\varepsilon^3 \xi^2}\left\{1 - k\left[\sqrt{t} + 2\int_0^t \xi(\tau)\dot{\xi}(\tau)\sqrt{(t-\tau)}\,\mathrm{d}\tau\right]\right\} \tag{30}$$

and the point denotes differentiation with respect to the dimensionless time. The boundary conditions for this integro-differential equation are

$$\xi(0) = 1, \quad \dot{\xi}(0) = \sqrt{\left(\frac{3/2}{1 + \dfrac{1}{30\varepsilon^6}}\right)}. \tag{31}$$

Knowing the solution $\xi(t)$ we are able to find the maximum value ξ_m at which $\dot{\xi} = 0$.

The value ξ_m, defined by equation (1), is a function of three parameters, namely k, Re and We, therefore a general formula for ξ_m is difficult to find. Several special cases have been analyzed numerically. For $k = 0 = Re^{-1}$ we have found

$$\xi_m = \sqrt{(We/3)} \tag{32}$$

if $We > 100$. For smaller values of the Weber number this formula is not true; e.g. for $We = 5$ we have found $\xi_m = 2.000$, and for $We = 20$, $\xi_m = 3.077$; other values are given in [1].

Likewise, for $k = 0 = We^{-1}$ the formula

$$\xi_m = 1.2941(Re + 0.9517)^{1/5} \tag{33}$$

gives good results. For $Re > 100$ this formula can be simplified thus

$$\xi_m = 1.2941 Re^{1/5}. \tag{34}$$

In the general case of $k = 0$, concerning the flattening of an impinging droplet without solidification, the value of ξ_m can be calculated from the following equation

$$\frac{3\xi_m^2}{We} + \frac{1}{Re}\left(\frac{\xi_m}{1.2941}\right)^5 = 1, \tag{35}$$

provided that $We > 100$, $Re > 100$.

The case with solidification ($k > 0$) has been studied numerically. The results are illustrated by the graph in Fig. 3. For the case $Re^{-1} = 0$, $We^{-1} = 0$ the numerical results may be represented by an approximate formula

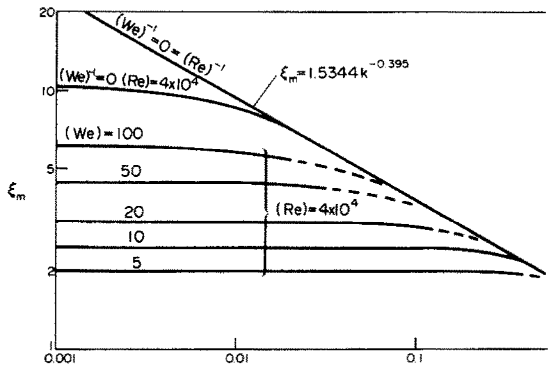

FIG. 3.

$$\xi_m = 1.5344 k^{-0.395}. \tag{36}$$

An extensive discussion of the solution is given in [1].

3. EXPERIMENTAL STUDY, CONCLUSIONS

In experiments the velocity of the droplet, and the area covered by it, can be measured without serious difficulties. The droplet diameter, however, if determined by weight is impossible to be measured since the mass of an alumina particle of $D = 2 \cdot 10^{-5}$ m is $1.3 \cdot 10^{-8}$ g. The method of diameter determination by weight can be applied only to droplets of greater diameters such as have been used in the present experimental study in which lead or tin droplets were thrown on a flat surface, in general inclined at the angle α from the horizontal. Most of the experimental points have been obtained for a horizontal surface ($\alpha = 0$) of various materials as wood, gypsum, plexiglas, copper, steel, aluminium. These 81 points are marked in Fig. 4 by circles. The range of the Weber numbers was 25–7300, that of the Reynolds numbers 11 000–150 000, that of the Péclet numbers 70–1300. In plasma spraying of alumina powder these values vary in the range $We = 1000$–$10\,000$, $Pe = 4000$–8000.

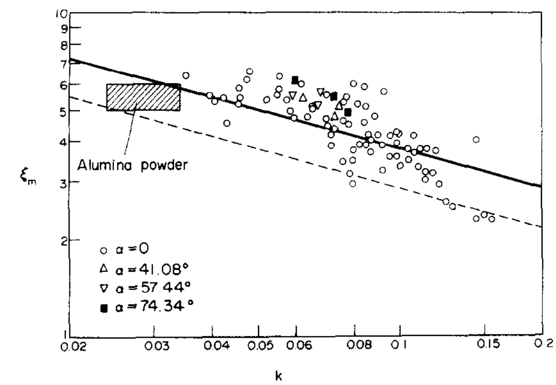

FIG. 4.

The accuracy of the experiments left much to be desired because it was difficult to maintain the droplet in the state of liquid at the temperature of fusion. Because of the great values of the Weber and Reynolds numbers an attempt was made to correlate the results in a form given by (36) as it is shown in Fig. 4. In order to calculate the number U, appearing in (2) and (28), one was obliged to decide which case of solidification should be chosen for the case considered. At hand was the equation, determining the quantity U, for the Stefan problem of solidification from a half-space of superheated liquid of temperature $T_p' > T_s'$, with T_s' as the temperature of fusion, on the surface of a half-space with λ_f, c_f, ρ_f and $a_f = \lambda_f/c_f\rho_f$ as the heat conductivity, specific heat, density, and thermal diffusivity of the mould, namely

$$\frac{U}{2}\sqrt{\pi} = \frac{T_0}{\left[\operatorname{erf}\frac{U}{2} + \sqrt{\left(\frac{\lambda c\rho}{\lambda_f c_f \rho_f}\right)}\right] e^{U^2/4}} - \frac{T_p \cdot \sqrt{\left(\frac{\lambda' c' \rho'}{\lambda c \rho}\right)}}{\operatorname{erfc}\left[\frac{U}{2}\sqrt{\left(\frac{a}{a'}\right)}\right] \cdot \exp\left(-\frac{U^2 a}{4a'}\right)}, \quad (37)$$

where
$$T_0 = \frac{\lambda(T_s' - T_0')}{a\rho\Delta h}, \quad T_p = \frac{\lambda(T_p' - T_s')}{a\rho\Delta h}, \quad (38)$$

and T_0' denotes the initial temperature of the mould, whereas Δh is the latent heat of fusion. Disregarding the superheat of the liquid $T_p = 0$ must be put. In this case the equation (37) is also valid for the isothermal case in which $\sqrt{(\lambda c\rho/\lambda_f c_f \rho_f)} = 0$.

The analysis of the experimental results has shown that the thermal properties of the cold surface, on which the droplets solidify, have essentially no influence on the value of ξ_m, and this fact has been observed as well for isolators as for good conductors as aluminium or copper. Although this result is not quite clear, we have used the value of U calculated from (37) for the isothermal case, i.e.

$$T_0 = \frac{\lambda(T_s' - T_0')}{a\rho\Delta h} = \sqrt{\pi}\frac{U}{2}\operatorname{erf}\frac{U}{2}e^{U^2/4}. \quad (39)$$

At $T_0' = 293$ K we have calculated $T_0 = 0.8084$ and $U = 1.1387$ for tin, $T_0 = 1.4848$ and $U = 1.4410$ for lead, $T_0 = 3.0962$ and $U = 1.8454$ for alumina Al_2O_3. The range of the numbers k was therefore 0.035–0.16 in the experiments, whereas for the alumina particles (Fig. 1) there was $k = 0.0238$–0.0337. In Fig. 4 the curve (36) has been drawn showing that the agreement between theory and experiments is not very bad. The shadowed area in Fig. 4 shows the range of data for Al_2O_3 droplets. In further study the influence of the angle α has been investigated. The photograph in Fig. 5 shows ten specimens of lead thrown on an aluminium

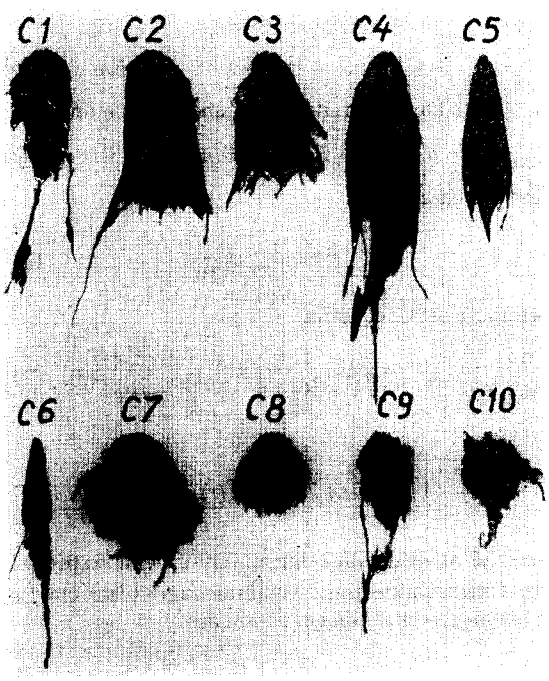

FIG. 5.

Table 1

Exp. No.	α (degrees)	D (mm)	w (ms^{-1})	A (mm^2)	We	Re	Pe	k	ξ_m
C1	57.44	3.955	3.76	316	1307.4	70824	614.7	0.0660	5.071
C2	57.44	4.769	3.90	548	1696.0	80580	699.3	0.0590	5.539
C3	57.44	3.811	3.70	384	1219.9	67156	582.8	0.0678	5.802
C4	74.34	4.649	3.89	657	1644.9	86130	747.5	0.0598	6.221
C5	74.34	3.362	3.70	274	1076.2	59244	514.2	0.0721	5.555
C6	74.34	2.887	3.70	162	924.1	50874	441.5	0.0779	4.974
C7	41.08	4.344	3.80	445	1466.7	78617	682.3	0.0626	5.480
C8	41.08	3.107	3.74	208	1016.2	55342	480.3	0.0746	5.238
C9	41·08	3.286	3.73	188	1069.0	58374	506.6	0.0727	4.709
C10	0	3.259	3.98	174	1207.1	61775	536.1	0.0707	4.567

surface. The data for these tests are given in Table 1. Although the "disks" for the case of the surface nearly vertical ($\alpha = 74.34°$) are elongated, the ratios ξ_m do not differ from those obtained for $\alpha = 0$. Previously it was thought fit to prepare a similar theory for one-dimensional flow of liquid, but the experimental results (Table 1) do not confirm it; this theory for $We^{-1} = 0 = Re^{-1}$ gives somewhat lower values of ξ_m represented in Fig. 4 by the dotted line.

Another theoretical attempt was made [1] to take into account the intensive cooling on the disk boundary. The theoretical corrections gave some small improvement in the agreement with experiments; however these corrections are not taken into account in the correlation in Fig. 4.

REFERENCE

1. J. Madejski, Solidification des gouttes projetées sur une surface froide, IBJ Internal Report (1975).

SOLIDIFICATION DE GOUTTELETTES SUR UNE SURFACE FROIDE

Résumé—Le problème annoncé dans le titre a été traité par voie théorique et expérimentale. La théorie utilise un modèle simple d'écoulement radial bidimensionnel. Le degré d'aplatissement d'une gouttelette ξ_m dépend des nombres de Weber, de Reynolds et de Péclet ainsi que de la constante de congélation U obtenue par résolution d'un problème de Stéfan. L'accord entre théorie et expérience n'est pas mauvais lorsque la constante U, dans le problème de Stéfan, est relative à une surface de refroidissement isotherme.

DAS ERSTARREN VON TROPFEN AUF EINER KALTEN OBERFLÄCHE

Zusammenfassung—Das Problem des Erstarrens von Tropfen auf einer kalten Oberfläche wurde theoretisch und experimentell untersucht. In der Theorie wird von dem einfachen Modell einer zweidimensionalen Radialströmung Gebrauch gemacht. Der Abflachungsgrad ξ_m eines Tropfens hängt von der Weber–, der Reynolds– und der Péclet–Zahl ab, sowie von der Erstarrungskonstanten U, die aus der Lösung des Stefan-Problems entnommen wird. Die Übereinstimmung der Theorie mit den Versuchsergebnissen ist befriedigend, wenn die Konstante U aus der Lösung für das Stefan-Problem mit isothermer Kühlfläche gewonnen wird.

ЗАТВЕРДЕВАНИЕ КАПЕЛЬ НА ХОЛОДНОЙ ПОВЕРХНОСТИ

Аннотация — Названная задача исследуется теоретически и экспериментально. В теории используется простая модель двумерного радиального течения. Степень выравнивания капли ξ_m зависит от чисел Вебера, Рейнольдса и Пекле и от постоянной замерзания U, взятой из решения задачи Стефана. Соответствие теории с экспериментом является неплохим, если постоянная U берется для задачи Стефана с изотермиечской охлаждаемой поверхностью.